

MA 555: Introduction to Manifold Theory

3 credit hours, Fall 2017

Instructor: Dr. Andrew Cooper, andrew.cooper@math.ncsu.edu

Class Meetings: MWF 11.45 am – 12.35 pm in 1218 SAS Hall

Office Hours: TBA in 3232 SAS Hall

Course Website: Moodle via wolfware.ncsu.edu

Prerequisites: MA 405, MA 426

Text: John Lee, *Introduction to Smooth Manifolds*, 2nd ed. (2012), \$99 via amazon.com

This book is available free electronically via NCSU Libraries.

Other resources: John Milnor, *Topology from the Differentiable Viewpoint*, rev. ed., \$44 via amazon.com;
Michael Spivak, *A Comprehensive Introduction to Differential Geometry*, 3rd ed. (1999).

Other materials: Internet- and PDF-capable device.

University Policies: Students are responsible for reviewing the PRRs which pertain to their course rights and responsibilities. These include: policies.ncsu.edu/policy/pol-04-25-05 (Equal Opportunity and Non-Discrimination Policy Statement), oied.ncsu.edu/oied/policies.php (Office for Institutional Equity and Diversity), policies.ncsu.edu/policy/pol-11-35-01 (Code of Student Conduct), and policies.ncsu.edu/regulation/reg-02-50-03 (Grades and Grade Point Average).

Accommodations: Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, students must register with Disability Services for Students at 1900 Student Health Center, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (REG 02.20.01)

OVERVIEW

Scope. Differential geometry is the business of using the notions of calculus (that is, differentiation and integration) to describe and detect the shapes of objects. In this course we will explore the differential geometry of smooth manifolds. The main thrust of the course is to learn how to carry out the computations of calculus on curved spaces. Along the way to this somewhat pedestrian end, we'll encounter some very deep mathematics.

The official course description is:

An introduction to smooth manifolds. Topics include: topological and smooth manifolds, smooth maps and differentials, vector fields and flows, vector bundles, tensors, differential forms, exterior calculus, and integration on manifolds.

We will cover this material according to the following (tentative) schedule. Chapters are from Lee:

smooth manifolds and their maps: (ch. 1-2) (1 week)

tangent vectors and vector fields: (ch. 3, 8) (2 weeks)

vector bundles: (ch. 10) (1 week)

tensors and tensor bundles: (ch. 12) (2 weeks)

cotangent bundle: (ch. 11) (1 week)

exterior calculus: (ch. 14) (2 weeks)

immersions and submersions: (ch. 4-5) (1 week)

flows and foliations: (ch. 9, 19) (1 week)

integration: (ch. 16) (1 week)

de Rham cohomology: (ch. 17) (1 week)

Student Learning Outcomes. A student who successfully completes this course will be able to:

- (1) Construct manifolds and tensor bundles.
- (2) Carry out tensor calculus computations, including: pushforward, pullback, tensor product.
- (3) Use vector fields to construct flows and submanifolds.
- (4) Determine whether coordinate expressions and identities are well-defined.
- (5) Distinguish between covariant and contravariant tensors.
- (6) Carry out exterior calculus computations, including: exterior product, exterior differentiation, integration of differential forms.
- (7) Prove Stokes' Theorem.

Prerequisites. In addition to the formal prerequisites, some students have reported finding MA 518 helpful.

CLASSROOM MEETINGS

For the most part lectures will be in the traditional format: I will lecture and you will take notes and ask questions. However, we will also spend time in active-learning activities such as worksheets, discussions, and on-the-spot problems.

Attendance at and active participation in all class meetings is essential.

ASSIGNMENTS AND GRADES

Evaluation. Your grade will be determined solely on the basis of the following graded work:

Homework: Homework will be due approximately once every two weeks. In total it will be worth 20% of your grade.

Exams: There will be two mid-term exams. Each exam will be worth 25% of your grade.

Final Exam: worth 30% of your grade.

Missed Assignments. If you will miss an exam or other assignment, please let me know *in writing* and *as soon as possible*. Assignments missed for valid reasons may be made up consistent with the University's policy on attendance, Regulation 02.20.03.

Homework. Each homework assignment will consist of three parts:

Readings: from the textbook and other resources.

Exercises: are for your practice and thought. They will not be collected, but should be considered mandatory.

Problems: will be submitted and graded.

Late homework will not be accepted except as described above.

Homework sets should be typed (preferably with L^AT_EX). See the course website for a list of L^AT_EX resources.

Exams. The first exam will be an in-class, closed-book, closed-notes exam. The second exam will be a take-home exam, which you will have about a week to complete. The final exam will be a three-hour exam.

Academic Integrity. The definition of academic integrity is simple and broad: *do not take credit for others' work*. This applies to all assignments. All assignments—absent an explicit statement to the contrary—should be completed individually.

You may not collaborate on exams in any form. You may not use any aids except those approved through the Disability Services Office and arranged with me in advance. This includes, but is not limited to: textbooks, crib sheets, electronic calculators, electronic communications devices, tattoos of formulæ and séances with the ghosts of C. F. Gauß, B. Riemann, D. Hilbert, and/or T. Levi-Civita.

Infractions of academic integrity will be addressed through the University's Office of Student Conduct pursuant to University Policy 11.35.01 and Regulation 11.35.02.

OUTSIDE OF CLASS

Office Hours: There is no need to make an appointment during my office hours listed above. In addition, I am also available by appointment (email is best).

Email: *All email announcements and correspondence will be sent to your official ncsu.edu email address.*