MA 591
PRINCIPLES OF GENERAL RELATIVITY

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DEPARTMENT OF MATHEMATICS

MA 591-004 — Fall, 2017

PRINCIPLES OF GENERAL RELATIVITY

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Grading:

Grades will be assigned based on participation in class. Two midterm tests and the final exam will be given, all of them home taken. The final exam can be replaced by a paper.

Course objectives:

- to overview Minkowski spacetime physics;
- to motivate transition to general relativity via an analysis of the equivalence principle;
- to introduce basic concepts of the manifold theory and differential geometry, with a stress on physical interpretation of differential geometric objects that are essential for describing gravity phenomena;
- to introduce Einstein equations and to trace their impact on physics in a curved spacetime (this includes the localization issue);
- to develop the variational formalism of general relativity (both Lagrangian and Hamiltonian) thus providing the firm basis for the canonical gravity quantization (if time permits).

Texts:

1. C. W. Misner, K. S. Thorne, J. A. Wheeler: "Gravitation", W. H. Freeman and Co., San Francisco (1998) (section numbers below refer to this source, unless stated otherwise).
2. B. F. Shutz: " A First Course in General Relativity", Cambridge University Press, Cambridge (2000)
3. R. M. Wald "General Relativity", The University of Chicago Press, Chicago and London (1984)
4. R. L. Bishop and S. I. Goldberg: "Tensor Analysis on Manifolds", Dover Publ., Inc., New York (1980)

Course Topics:

1. Minkowski spacetime. Vectors and forms. Tensors, exterior forms, polyvectors (2.1–2.7).

2. Minkowski spacetime metric. Interval, light cone, inertial frames and hyperbolic transformations (2.9).

3. Lorentz contraction and twin paradox in Minkowski spacetime. Accelerating and rotating observers, Fermi–Walker transport (6.1–6.6).

4. Energy–momentum tensor, conservation of energy–momentum and equations of motion. Perfect fluid (5.1–5.2, 5.5–5.10).

5. Doppler shift. Red shift in a gravity field. Insufficiency of Minkowski space. Equivalence principle (7.1–7.5).

6. Manifolds, local trivializations, tangent spaces, tangent bundles. Vector fields, differential forms, exterior calculus (9.1–9.7).

7. Connections on manifolds and covariant derivatives. Connections over maps. Pseudoriemannian metric, Levi Civitta connection (10.1–10.5, 5.7–5.10(B)).

8. Curvature, Riemann tensor, symmetries of the Riemann tensor. Geodesic deviation as manifestation of gravity (11.1–11.6, 13.1–13.5).

9. Bianchi identities and contracted Bianchi identities. E. Cartan moment of rotation (15.1–15.4).

10. Einstein equations. Plausibility arguments and variational principle (16.1–16.5, 17.1–17.4).

11. Newtonian limit and correspondence principle. Metric and gravitational potential. New features of gravity in general relativity. Gravitational degrees of freedom. Einstein equations and contracted Bianchi identities (15.5–15.6).

12. Weak field approximation. Gravity waves in the linearized theory of gravity. Radiation reaction. Insufficiency of the weak field approximation (18.1–18.3).

13. Schwarzschild geometry. Centrally symmetric solution of Einstein equations. Pit in the potential as a key feature of general relativistic centrally symmetric gravity field (23.1–23.3).

14. Homogeneous cosmologies. Friedmann model, Robertson–Walker models. Matter dominated and radiation dominated Universe. Cosmological red shift (27.1–27.10).

15. Lagrangian variational formalism, Palatini method (21.2–21.3), $3 + 1$ split of spacetime (21.4), Hamiltonian formalism, general relativity as geometrodynamics, the initial value problem (21.6–21.11), the Einstein–Hamilton–Jacobi equation (43.1–43.3).