BMA/MA 774, Partial Differential Equation Modeling in Biology

FA 17, TR 1:30-2:45, SAS 2225
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Office hours MW 2-3, SAS 4226, unless otherwise announced

Modeling with and analysis of partial differential equations as applied to real problems in biology. Review of diffusion and conservation laws. Waves and pattern formation. Chemotaxis and other forms of cell and organism movement. Introduction to solid and fluid mechanics/dynamics. Introductory numerical methods. Scaling. Perturbations, asymptotics, Cartesian, polar and spherical geometries. Case studies.

BMA/MA 774 is an introduction to linear and nonlinear PDE as models for biological systems. Students should have some modeling background, and must have had ODEs. The official MA 401 or 501 requirement is out of date. MA 401 or 501 would be useful, but the first month or so covers the basics of linear PDE as might be seen in 401/501. The rest of the course develops a toolkit for modeling with nonlinear PDE and solving them. Typically the class is a mixture of students from BMA, BME, MA, AMA, and occasionally others such as OR. Be aware that this class is only offered every two years or so.

Goals: A student who successfully completes this course will be able to model a spatially-dependent biological phenomenon with a system of PDE, carefully define all terms, explain the model, classify the system, identify suitable methods of solution, implement basic solution methods, interpret the results and their biological significance, clearly present the results both visually and verbally, and, if appropriate, revise the model.

Prerequisites: ODE and eigens: MA 341 or BMA 771 or similar. Modeling: BMA 567 or 771 or similar (such as MA 573 or 574). Intellectual maturity and independence: graduate status.

Textbook: Datta and Rakesh, *An Introduction to Modeling of Transport Processes: Applications to Biomedical Systems* (2010). Available new and used, and the NCSU libraries have it as an e-book. We will also use additional selected readings (either handouts or on reserve).

Grades: Grades will be determined by homework sets, a term project, and a final exam. In keeping with the graduate status of this class, the homeworks will be non-routine.

FAQ

Q: How does this class compare with other classes available?
A: MA 401, 501, 534, and 734 are only about linear PDE and methods for linear PDE. Most of the problems in biology will require nonlinear models, and the linear methods are in general not useful for them. That is why we only briefly cover the linear methods in BMA 774. MA 584, 587, and 788 cover numerical methods, but do not cover modeling. Many of the topics in BMA 774 are covered in MAE courses such as 308, 310, 314, 355, 356, 455, 505, 550, 557, 560, etc. and PY courses such as 463, but BMA 774 is specifically focused on biological problems (many of which do not involve mechanics) and the subtleties of modeling them.

Q: Do I need to know Maple and Matlab and Comsol already?
A: They will all be introduced in class. How much we cover depends on the background of the students in the class.

Topics (depending on time and student background and interests)

anatomy of a PDE
diffusion equation, derivation from random walk
dimensional analysis
separation of variables and analytical solutions
boundary and initial conditions
diffusion on infinite domain
Maple, Matlab, Comsol
forward Euler method; finite element method
conservation equations
advection; advection-diffusion equation
nonlinear PDE: Fisher-Kolmogorov equation
coupled PDE
force balance equations; Navier-Stokes equations
porous media
simple examples of fluid flow (pipe flow, Reynolds number, Stokes flow, flow around a sphere)
chemotaxis
pattern formation
radial coordinates; Krogh cylinder
modeling growth