MA 755: Introduction to Riemannian Geometry
3 credit hours, Spring 2019

Instructor Dr. Irina Kogan, iakogan@ncsu.edu

Class Meetings MW 11:45 am – 1:00 pm

Prerequisites MA 555.

Text John Lee, *Riemannian Manifolds*, (1997).

Course objective

To learn how to define and compute main geometric characteristics of a differential manifold with metric.

Catalog description

> An introduction to smooth manifolds with metric. Topics include: Riemannian metric and generalizations, connections, covariant derivatives, parallel translation, Riemannian (or Levi-Civita) connection, geodesics and distance, curvature tensor, Bianchi identities, Ricci and scalar curvatures, isometric embeddings, Riemannian submanifolds, hypersurfaces, Gauss Bonnet Theorem; applications and connections to other fields.

Approximate schedule

(Chapters are from Lee)

review of tensors, manifolds, and tensor bundles (ch. 2) (1 week)

Riemannian, pseudo-Riemannian and sub-Riemannian metrics. (ch. 3) (1 week)

connections, covariant derivatives, parallel translation. (ch. 4) (2 weeks)

Riemannian (or Levi-Civita) connection, geodesics, normal coordinates. (ch.5) (2 weeks)

Midterm Exam. Monday, February 18th, 2019.

geodesics and distance (ch. 6) (2 weeks)

curvature tensor, Bianchi identities, Ricci and scalar curvatures, Einstein equations. (ch. 7) (2 weeks)

Riemannian submanifolds, hypersurfaces in the Euclidean space. (ch. 8) (2 weeks)

the Gauss-Bonnet theorem. (ch.9) (2 weeks)

applications (1 week)

Student Learning Outcomes

A student who successfully completes this course will be able to:

1. State the definition of a metric and an isometry.
2. Use metrics to define arc-length, distance and volume.
3. Give proofs of and use the Divergence Theorem and Green's Theorem on Riemannian manifolds.
4. State definitions and properties of geometric invariants of Riemannian manifolds: Riemannian curvature, Ricci curvature, scalar curvature.
5. Define and compute Riemannian connection, covariant derivatives, and parallel translation.

6. Give a definition of geodesics (as curves with zero acceleration) and write their defining equations. Prove that geodesics are locally length-minimizing curves.
7. State Hopf-Rinow completeness theorem and explain its relationship between completeness and global length-minimizing properties of geodesics.
8. State definitions and properties of geometric invariants of Riemannian submanifolds and hyperserfaces.
9. Distinguish between intrinsic and extrinsic invariants of Riemannian submanifolds.
10. State the Gauss-Bonnet Theorem. Explain the main ideas behind its proof and its importance as a global-local result.

Course structure

For the most part, lectures will be in the traditional format. However, some time will be spent in active-learning activities such as worksheets, discussions, and on-the-spot problems. Attendance at all class meetings is essential.

Assignments and Grades

Evaluation

The standard 10 point scale with +/- grading will be used. The grade will be determined on the basis of the following graded work:

Homework: will be due approximately once every 7 -10 days. In total it will be worth 50% of the course grade.

Exams: there will be one mid-term exam worth 25% of the course grade.

Final Exam: is worth 25% of the course grade.

Missed/Late Assignments

In the case of missed exam or assignment, please, let the instructor know *in writing* and *as soon as possible*. Assignments missed for valid reasons may be made up consistent with the University's policy on attendance, Regulation 02.20.03. `policies.ncsu.edu/regulation/reg-02-20-03`. Late homework will not be accepted without a valid reason.

Homework

Each homework assignment will consist of three parts:

Reading appropriate sections of the text-book is essential.

Exercises are for your practice and thought. They will not be collected, but should be considered mandatory.

Problems will be submitted and graded.

Homework sets should be typed (preferably with LaTeX). See the course website for a list of LaTeX resources.

Statement for students with disabilities

Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, students must register with Disability Services for Students at 1900 Student Health Center, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (REG 02.20.01)

N.C. State University Polices, Regulations, and Rules (PRR)

Students are responsible for reviewing the PRRs which pertain to their course rights and responsibilities. These include: http://policies.ncsu.edu/policy/pol-04-25-05 (Equal Opportunity and Non-Discrimination Policy Statement), http://oied.ncsu.edu/oied/policies.php (Office for Institutional Equity and Diversity),http://policies.ncsu.edu/policy/pol-11-35-01 (Code of Student Conduct), and http://policies.ncsu.edu/regulation/reg-02-50-03 (Grades and Grade Point Average).