Syllabus

Contents

2.1 Instructor and Basic Course Information

Instructor:	Dr. Bevin Maultsby
Moodle Page:	https://moodle-courses1819.wolfware.ncsu.edu/course/view.php?id=4750
Communication:	We will use Piazza for class discussion and homework questions. You are encouraged to use Piazza to discuss concepts and homework (with discretion–hints and suggestions are okay, but full solutions will be deleted). You can post under your name or you may be anonymous to your classmates (however, I will be able to see who you are). Please be respectful of your fellow classmates' ideas and attempts. Enroll in our class Piazza through Moodle. Ideally, math questions should be asked in class, in office hours, or on Piazza. In general, I will not answer math questions sent by email–if you have a question you would like to send to me, please post it on Piazza. I will monitor and respond to questions on Piazza. Email may be used for confidential and private discussions about grades, scheduling office hours, etc. In general, I do not respond to email outside of business hours (M–F, 9am-5pm). It may take 24-72 hours for me to respond to an email.
Office Hours:	– Mondays and Thursdays, 3:15-4:15pm – Thursdays, 12:00-1:00pm – by appointment
Office:	SAS Hall 3230
Class:	MWF, 11:45am-12:35pm in 105 Ricks Hall
Instructor Contact:	bmaults@ncsu.edu, 919-515-1876 (no voicemail)

2.2 Course Prerequisites or Restrictive Statements.

Prerequisite: MA 425 and MA 405.

Credit is not allowed for both MA 591 and MA 512.

2.3 GEP Designation

None: this class does not satisfy a GEP requirement.

2.4 Student Learning Outcomes

The material covered in this course builds upon the rigorous foundation established in MA 425. We will explore many concepts, definitions, and theorems in $\mathbb{R}^n$ and general metric spaces; some of the material will be extensions of material from 425, while other material will be new.

Students in this course are expected to...

1. Study the background spaces we work in and the topology of subsets of these spaces.
   (a) Identify different types of ambient spaces: Euclidean spaces, metric spaces, normed vector spaces, inner product spaces.
   (b) Determine if a subset of Euclidean space is open or closed (or both or neither) using the definition of these notions, as well as relevant theorems about interior points, boundary points, accumulation points, closure, sequences, and completeness.
   (c) Identify when a set is compact using the "open covers" definition, as well as the definition of sequential compactness and the Heine-Borel theorem.
   (d) Identify when a set is connected or path connected.
2. Study vector-valued functions (e.g. functions $f : \mathbb{R}^m \to \mathbb{R}^n$)
   (a) Define continuity, and identify when a function is continuous using theorems.
   (b) State and use the major theorems pertaining to continuous functions on compact sets.
   (c) State and use the Intermediate Value Function.
   (d) Characterize uniform continuity.
   (e) Define what it means to say that a function is differentiable.
   (f) Represent the derivative of a function $f : \mathbb{R}^m \to \mathbb{R}^n$ as a matrix, and understand how to apply linear algebra to better understand what the derivative represents as a linear transformation.
   (g) Establish higher-dimensional versions of the Chain Rule, Product Rule, the Mean Value Theorem, amongst others.
   (h) Learn Taylor's Theorem and use it to classify extrema.
   (i) State and outline the main idea of the proofs for the Inverse Function Theorem and the Implicit Function Theorem.
3. Extend our knowledge of integration from MA 425 to integration of functions $f : \mathbb{R}^n \to \mathbb{R}$.
   (a) Create partitions of domains $A \subset \mathbb{R}^n$, which will more varied than intervals $[a, b] \subset \mathbb{R}$.

(b) Understand Lebesgue's Theorem giving us conditions for integrability.

(c) Use Fubini's Theorem in many cases to actually compute Riemann integrals as iterated integrals.

(d) Use the Change of Variables theorem to generalize the idea of "u-substitution" to higher dimensional domains.

4. Time permitting, additional topics will include the Generalized Stokes Theorem (and its specific variations) and Contraction Mapping.

In general, we will prove almost every result we cover. However, certain results (notably, the Inverse Function Theorem and the Change of Variables Theorem) would take hours to prove in full detail. Thus we give "plausibility arguments" for those results only.

Students will practice these concepts using the Moodle quizzes, discuss and expand their understanding of these concepts on the homework, and demonstrate mastery of these concepts on the examinations.

It is expected that Midterm 1 will cover material from 1(a)-2(d) and Midterm 2 will cover 2(e)-3(b). The final will be cumulative and will cover all material from the course.

Please note that this section of MA 591 is a sequel to MA 425; the level of effort and expectation of rigor is similar to what you should have encountered in MA 425. On homework and examinations you will be expected to write many proofs, including the standard "$\varepsilon - \delta$" style proofs you used in MA 425.

Students who are enrolled in MA 591 rather than 591 will have three extra Moodle videos + quizzes to complete, see Moodle.

2.5 Student Expenses

2.5.1 Textbook information

To do well in Real Analysis, you should expect to spend significant time reading and rereading the textbook. Please see Moodle for a list of posted errrata/typos (from the textbook authors).

Required: *Elementary Classical Analysis* by Marsden and Hoffman, 2nd Edition.
ISBN: 9780716721055

Optional: In addition to the assigned book, you may find the following books helpful

- *Understanding Analysis* by Stephen Abbott (Recommended for Chapter 3 (Topology of $\mathbb{R}$) if you want more examples on $\mathbb{R}$.)

  On Reserve at: D.H. Hill Course Reserves

- *A First Course in Real Analysis* by Murray H. Protter, Charles B. Morrey, Jr. (second half of

the book)

On Reserve at: D.H. Hill Course Reserves

2.5.2 Other expenses

Not applicable.

2.6 Course Overview (Catalog Description)

Calculus of several variables, topology in n-dimensions, limits, continuity, differentiability, implicit functions, integration.

2.7 Course Structure

In general this course will consist of three lectures per week (MWF). Lectures will be interactive and students are encouraged to come to class prepared for discussions and activities.

Quizzes. MA 591 is an exceptionally challenging course for most students. **Reading the material in the book is required for success.** Therefore, there are ∼20-minute textbook-based quizzes on Moodle. Quizzes will require you to demonstrate knowledge of definitions and theorems. You may be required to figure out a quick proof on your own to get to the right answer–therefore, you should only attempt a quiz when you can give it your full attention. Although the quizzes are open notes/open book, your success in the course is closely linked to your familiarity with the terminology; heavy dependence during the quizzes on notes/books is discouraged.

Quizzes are divided up by chapter sections, not by week. Thus, there may be more than one due in a given week. You can see the open/close dates of the quizzes in the Calendar block on Moodle; you are responsible for tracking when quizzes are due. In general, most quizzes close on Friday at 11pm.

Homework. Homework has three purposes: (1) to better understand the mathematical concepts in the course, (2) to improve your mathematical writing, and (3) to improve your ability to discuss difficult concepts with your peers.

Some homework questions ask you to demonstrate mastery of the concepts by using relevant theorems to perform calculations–these problems should be straightforward and do not require formal writing. Other homework questions will ask for (rigorous) proofs, which will be evaluated not only on mathematical correctness but also on the clarity of your writing.

You will submit each homework set with 1-3 other students (required). Homework sets may require many days of thought, and you should be working on them and discussing them as soon as possible. Homework assignments will be posted on Moodle and are due by the start of class on the listed day.

Homework must be submitted online as a PDF via the link posted on Moodle. Homework groups may typeset their homework in LaTeX(and submit the PDF, not the tex file) or scan your homework and turn it into a PDF. You may find "Save as PDF" or "Print to PDF" useful. Students also use the app CamScanner–you do not need to buy it to remove the watermark. Homework will be graded holistically–your grade will reflect both your mathematical correctness and your writing.

(For those of you who are interested in typesetting your homework in LaTeX, see the guide posted on Moodle. If everyone in your homework group is comfortable with LaTeX, you may look into sharing .tex files on Overleaf, or via google drive.)

Note: Microsoft Word is not a good typesetting program for mathematics.

Class expectations. Any student using a laptop, phone, iPod, or other device irrelevant to class will be asked to put it away or leave. During tests, cell phones must off and out of sight. Being on a device of any kind will cause you to essentially be marked absent (you will be marked "On Device," which will count toward your total number of unexcused absences.)

2.8 Weekly Course Schedule

The course schedule is subject to change. Any changes to the class schedule will be announced on Moodle. Do not unsubscribe from the Announcements forum.

2.8.1 List of topics

Here are the major topics for MA 591, numbered by corresponding textbook chapter(s):

1. Spaces (Euclidean, metric spaces, normed vector spaces, inner product spaces)

2.-3. Topology (open, closed, compact, connected, and relevant ideas)

4. Continuous mappings

6. Differentiation in higher dimensions

7. Inverse and Implicit function theorems

8.-9. Integration

   * Additional material (time permitting) may include the generalized Stokes' theorem (and its specific variants) and contraction mapping.

The largest chapter is Chapter 6 and we will spend several weeks covering differentiation.

2.8.2 Projected schedule of assignments, quizzes, and tests

Topics. This schedule may change as some topics can go faster or slower than predicted.

Date	Sections	Topics
Jan. 7-11	1.6-1.7	Introduction to the course. Euclidean space. Norms, inner products, and metrics. These concepts give us a way to take certain geometric notions we enjoy about Euclidean spaces (namely lengths, angles, distances) to other settings. Beginning of Ch. 2.
Jan. 14-18	2.1-2.6	The basic topology of open and closed sets, and the related notions of the interior of a set, the accumulation points of a set, the closure of a set, and the boundary of a set.
Jan. 23-25	2.7-3.1	Sequences–review from 425. The new material pertains to using sequences to characterize closed sets. The open cover definition of compactness. This notion is a hard one for students. *Note that 1/21 is Martin Luther King Jr. Day, and we will not have class.*
Jan. 28-Feb. 1	3.1-3.5	Compactness continued, and the Heine-Borel Theorem... which applies in Euclidean space, but not in general! The Nested Set Property. Connectedness, including path-connectedness. Beginning of Ch. 4.
Feb. 4-8	4.1-4.5	Continuity, and the knowledge we get when a continuous mapping is over a compact or connected domain. Properties of continuous mappings, and the Intermediate Value Theorem. Uniform continuity.
Feb. 11-15	6.1, Test Review	Definition of the derivative. Review for Test 1.
Feb. 18-22	Test 1, 6.2-6.3	The first midterm will take place during class on Monday. This test will cover the above sections, with the exception of 6.1. The matrix representation of a derivative, and interpreting the deriative as a linear transformation.
Feb. 25-March 1	6.4-6.6	Conditions for differentiability, the Chain Rule, Tangent planes. Key notion: the chain rule says that the derivative of a composition is a matrix product–make sure you understand why.
March 4-8	6.7-6.9	The Mean Value Theorem. We will use Taylor's Theorem together with a supplement on Sylvester's Criterion to study maxima and minima. We will not have class on 3/8, but there will be a **required** posted video lecture.
March 11-15		*Spring Break – University closed*
March 18-22	7.1 & 7.2	The Inverse and Implicit function Theorems tell us when we may expect to be able to invert a function, or solve for one variable in terms of others (e.g. passing from $F(x,y)=0$ to $y=f(x)$).
March 25-29	8.1-8.3	We will begin integration for $f : A \subset \mathbb{R}^n \to \mathbb{R}$. Volumes and sets of measure zero. When is a function integrable?

April 1-5	8.3, 8.4, 9.2, Review	Fubini's Theorem gives us practical ways to evaluate integrals. Review for Test 2.
April 8-12	Test 2, 9.3	The second midterm will take place on Monday. The Change of Variables theorem (substitution) will likely not be proven entirely but will be discussed using plausibility arguments. We will revisit linear transformations and determinants from Linear Algebra. Examples will include polar coordinates and spherical coordinates.
April 15-17	9.3, Supplement	Time permitting, we will discuss line and surface integrals, revisiting Green's Theorem, Stokes' Theorem, and the Divergence Theorem from multivariable calculus. This progression will culminate in the Generalized Stokes Theorem. We will likely move through this material fairly quickly and proofs will be omitted. *Note that there is no class on 4/19.*
April 22-26	*Dead week*	We may visit contraction mapping, otherwise much of this week will be dedicated to review for the final exam.
April 29[1]	Final Exam	8–11am, usual classroom

Homework schedule. Writing advanced calculus proofs is challenging for students and takes a lot of time and energy. You should be sure to begin the next homework set as soon as possible.

A tentative schedule of homework assignments is provided below; occasionally due dates may be adjusted. You are responsible for checking Moodle regularly to make timely progress through the assignments. All homework sets are due at the beginning of class on the listed day.

Homework Set	Due Date	Homework Set	Due Date
HW 1	1/16	HW 6	3/4
HW 2	1/28	HW 7	3/25
HW 3	2/6	HW 8	4/5
HW 4	2/15	HW 9	4/22
HW 5	2/25		

2.8.3 Required field trips

Not applicable.

[1] Typos can occur. You are responsible for establishing your own final exam calendar using the website `https://studentservices.ncsu.edu/calendars/exam/`.

2.8.4 Laboratory, studio, or problem sessions

Not applicable.

2.9 Statement on Transportation

Not applicable.

2.10 Statement on Safety and Risk Assumption

Not applicable.

2.11 How Grades are Determined

Your grades are entirely based on your performance on quizzes, homework, and examinations–no special considerations are taken in account whatsoever. Your grades will be posted on Moodle.

2.11.1 The relative value of the various evaluation components of the course

Homework	20%
Quizzes	10%
Midterm 1	20%
Midterm 2	20%
Cumulative Final	30%

Attendance is taken but does not directly factor into your course grade. However, if you have **fewer** than 5 total absences from regular lectures, the final exam will replace your lowest midterm score (if beneficial). Being on a device of any kind will cause you to essentially be marked absent (you will be marked "On Device"); therefore, if you attend every day but are frequently on your phone, you will forfeit the chance to have your final replace your lowest midterm.

This policy is strict–regardless of circumstances, if you miss more than 4 classes you are losing out on valuable classroom time.

2.11.2 The conversion system from numerical to letter grading

The final grade will be assigned using the plus/minus grading system according to this chart:

A+ : 98–100	A : 93–97.99	A– : 90–92.99
B+ : 88–100	B : 83–87.99	B– : 80–82.99
C+ : 78–100	C : 73–77.99	C– : 70–72.99
D+ : 68–100	D : 63–67.99	D– : 60–62.99
	F : 0–59.99	

2.12 Late Assignments

Late homework shall be accepted with a penalty of 5 points per day (e.g., turning a Monday assignment in on Wednesday incurs a 10 points penalty, a Friday assignment handed in on Monday incurs a 15 point penalty). Be sure to begin each assignment early.

2.13 Attendance/Absence Policy

Please consult `https://policies.ncsu.edu/regulation/reg-02-20-03-attendance-regulations/`.

Missed regular classes. Attendance is taken but does not directly factor into your course grade; see §2.11. If you miss a regular class period, you are responsible for obtaining information that you may have missed from your classmates.

Missed midterms. A student who misses a midterm and accumulates fewer than five regular class absences will have their midterm grade of 0 replaced by their final exam score.

In the event that a student misses a midterm and accumulates five or more absences throughout the semester, the student will either (a) incur a 0 on the exam which will factor into the student's semester average, or (b) provide documentation of an excused absence through NCSU Class Absence Verification, see `https://dasa.ncsu.edu/students/absence-verification-process/`. Therefore, if you are sick on the day of a midterm, I recommend that you immediately begin the documentation process with an absence verification officer.

The midterm policy is strict–**there are no "makeup midterms."**

2.14 Academic Integrity

For the NCSU policy, see the Code of Student Conduct at `https://policies.ncsu.edu/policy/pol-11-35-01/`

Quizzes, Exams & Final: Students will neither give nor receive any assistance on any quiz, midterm, or the final exam. Use of cell phones and calculators during exams is not permitted.

Homework: You are allowed to work together on homework assignments; however, each student must turn in his or her own WebWork.

Utilization of the Honor Pledge: All exams and tests must be completed independent of assistance from other people. Implicit in any submission is the pledge that "I have neither given nor received unauthorized aid on this test or assignment."

Penalty for violations: Any violation of this policy will be reported to the Office of Student Conduct with a recommendation of a failing grade for the assignment and/or course.

2.15 Statement for students with disabilities

Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, students must register with the Disability Services Office at Suite 2221, Student Health Center, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (REG02.20.01) (`https://policies.ncsu.edu/regulation/reg-02-20-01/`).

2.16 Electronic Course Components

Please be advised that there are several resources available to assist students with technical or computer issues. Please consult `https://oit.ncsu.edu/help-support`.

This course will be recorded for current and potential future educational purposes. By your continued participation in this recorded course, you are providing your permission to be recorded.

2.16.1 Electronically hosted course components

The course site for the class is hosted on Moodle, which includes the weekly quizzes. You must address the accessibility of this website for you during the course drop/add period.

2.16.2 Privacy or accessibility

Instructor is not responsible for ensuring privacy or accessibility of electronic materials that are not required components of the course (e.g., links to supplemental information that is not part of the required reading list).

2.16.3 Required statement:

"Students may be required to disclose personally identifiable information to other students in the course, via electronic tools like email or web-postings, where relevant to the course. Examples include online discussions of class topics, and posting of student coursework. All students are

expected to respect the privacy of each other by not sharing or using such information outside the course."

2.17 Your Rights and Responsibilities

Students are responsible for reviewing the NC State University PRR's which pertains to their course rights and responsibilities:

- Equal Opportunity and Non-Discrimination Policy Statement
  https://policies.ncsu.edu/policy/pol-04-25-05
  with additional references at https://oied.ncsu.edu/equity/policies/
- Code of Student Conduct https://policies.ncsu.edu/policy/pol-11-35-01
- Grades and Grade Point Average https://policies.ncsu.edu/regulation/reg-02-50-03
- Credit-Only Courses https://policies.ncsu.edu/regulation/reg-02-20-15
- Audits https://policies.ncsu.edu/regulation/reg-02-20-04