

MA 798 Special Topics In Numerical Analysis: Inverse Problems

Lecture details

Lecture time/location: 1:30–2:45 Monday/Wednesday, SAS2106
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Office Hours: Thursday 1:00–2:00PM or by appointment

Course Description

Mathematical models of physical systems are often described by systems of differential equations. Given a complete description of a physical system, we can solve the governing differential equations to predict the outcome of some physical measurements; this is called the *forward problem*. In an *inverse problem*, we use the result of some measurements/observations to reconstruct/infer the values of the parameters that characterize the physical system and appear in the governing differential equations as coefficients, boundary data, initial conditions, etc.

This course provides an introduction to inverse problems that are governed by partial differential equations (PDEs) and methods for their numerical solution. The focus will be on variational formulations, ill-posedness, regularization, discretization methods, and optimization algorithms for large-scale inverse problems. In addition to learning about the theoretical aspects of inverse problems, the students will develop numerical implementations to gain insight into the effect of data noise, regularization, the choice of the parameter field, and the nature of the underlying PDE model on the identifiability of the model parameters. While our focus will be mainly on deterministic inverse problems, the course will also provide an introduction to the statistical formulations. Specifically, we will consider the Bayesian formulation of an inverse problem and highlight the connections between the deterministic and Bayesian perspectives of inverse problems. Model problems will be drawn from different areas of science and engineering, including image processing, continuum mechanics, and geophysics.

Grading Policy

The grading will be assigned on a 10-point scale: **A: 90 – 100, B: 80 – 89, C: 70 – 79, D: 60 – 69, F: \leq 60**. The cutoffs for the +/- grades are determined at the end of the semester. Your final grade in this course will be determined by marks earned on about six homework involving a mix of theory and computational experiments/implementation.

Prerequisites

Solid background in linear algebra (e.g., at the level of MA 405). Students should have also taken at least one course in differential equations (e.g., MA 341), and have some basic familiarity with PDEs. Background in numerical linear algebra (e.g., at the level of MA 580) is desirable. However, the required background in numerical methods will be covered as needed, albeit quickly. Also, the required background material on nonlinear optimization, variational methods, and basic probability will be covered throughout the course. Generally, a mathematically mature student will be able to acquire from the lectures the necessary mathematical and computational background. For computational aspects of the course, students should be familiar with programming in either Matlab or Python.

Course topics

Introduction to inverse problems with PDEs; ill-posedness and regularization; variational methods, weak forms; PDE-constrained optimization problems; computing derivatives via adjoints for steady and unsteady PDE problems; descent methods from nonlinear optimization, Newton-conjugate gradient method; Bayesian approach to inverse problems.

Useful references There is no required textbook, but several good references for variational inverse problems (which go far beyond the material covered in class) include:

- Theory and computational methods for inverse problems:
 - Heinz Engl, Michael Hanke, and Andreas Neubauer, *Regularization of Inverse Problems*, Dordrecht, 2nd edition, 1996. (classic book on regularization of inverse problems)
 - Curtis R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002. (focus on numerical solution methods, applications mainly in image restoration)
 - Kazufumi Ito, and Bangti Jin. *Inverse problems: Tikhonov theory and algorithms*. 2015.
- Numerical optimization background:
 - Jorge Nocedal and Stephen J. Wright, *Numerical Optimization*, Springer-Verlag, 1999.
 - C. Tim Kelley, *Iterative Methods of Optimization*, SIAM, 1999. (unconstrained, lots of practical advice, PDF is available online)
 - Phillip E. Gill, Walter Murray and Margaret H. Wright, *Practical Optimization*, Emerald, 1982.
- Optimization of systems governed by PDEs:
 - Max D. Gunzburger, *Perspectives in Flow Control and Optimization*, SIAM, 2003. (more general than implied by title)
 - Fredi Tröltzsch, *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*, Graduate Studies in Mathematics Vol. 112, AMS, 2010.
 - M. Hinze, R. Pinnau, M. Ulbrich, and S. Ulbrich, *Optimization with PDE constraints*, Springer, 2009.
 - Alfio Borzi and Volker Schulz, *Computational Optimization of Systems Governed by Partial Differential Equations*, SIAM, 2012.
- Probabilistic approach to inverse problems:
 - Albert Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, 2005. (statistical perspective; freely available on the web)
 - Jari Kaipio and Erkki Somersalo, *Statistical and Computational Inverse Problems*, Springer, 2005.
 - Tim Sullivan, *Introduction to Uncertainty Quantification*, Springer, 2016.
 - Masoumeh Dashti and Andrew M. Stuart. *The Bayesian Approach To Inverse Problems*. *Handbook of Uncertainty Quantification*, 2017. (arXiv preprint:<https://arxiv.org/abs/1302.6989>)

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Both faculty and students at North Carolina State University have a responsibility to maintain academic integrity. An informational brochure about academic integrity is available from the university and students are encouraged to obtain a copy.

"Academic dishonesty is the giving, taking, or presenting of information or material by a student that unethically or fraudulently aids oneself or another on any work which is to be considered in the determination of a grade or the completion of academic requirements or the enhancement of that student's record or academic career." (NCSU Code of Student Conduct)

Scholarly activity is marked by honesty, fairness and rigor. A scholar does not take credit for the work of others, does not take unfair advantage of others, and does not perform acts that frustrate the scholarly efforts of others. The violation of any of these principles is academic dishonesty. Penalties for a violation: For the first violation, you will receive a zero for your work and be put on academic integrity probation for the remainder of your stay at NCSU. The second violation may result in your suspension from NCSU. Both situations will involve the Office of Student Conduct.