MA 798 Special Topics In Numerical Analysis: Inverse Problems

Lecture details
Lecture time/location: 1:30–2:45 Monday/Wednesday, SAS2106
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Office Hours: Thursday 1:00–2:00PM or by appointment

Course Description
Mathematical models of physical systems are often described by systems of differential equations. Given a complete description of a physical system, we can solve the governing differential equations to predict the outcome of some physical measurements; this is called the forward problem. In an inverse problem, we use the result of some measurements/observations to reconstruct/infer the values of the parameters that characterize the physical system and appear in the governing differential equations as coefficients, boundary data, initial conditions, etc.

This course provides an introduction to inverse problems that are governed by partial differential equations (PDEs) and methods for their numerical solution. The focus will be on variational formulations, ill-posedness, regularization, discretization methods, and optimization algorithms for large-scale inverse problems. In addition to learning about the theoretical aspects of inverse problems, the students will develop numerical implementations to gain insight into the effect of data noise, regularization, the choice of the parameter field, and the nature of the underlying PDE model on the identifiability of the model parameters. While our focus will be mainly on deterministic inverse problems, the course will also provide an introduction to the statistical formulations. Specifically, we will consider the Bayesian formulation of an inverse problem and highlight the connections between the deterministic and Bayesian perspectives of inverse problems. Model problems will be drawn from different areas of science and engineering, including image processing, continuum mechanics, and geophysics.

Grading Policy
The grading will be assigned on a 10-point scale: A: 90 – 100, B: 80 – 89, C: 70 – 79, D: 60 – 69, F: ≤ 60. The cutoffs for the +/- grades are determined at the end of the semester. Your final grade in this course will be determined by marks earned on about six homework involving a mix of theory and computational experiments/implementation.

Prerequisites
Solid background in linear algebra (e.g., at the level of MA 405). Students should have also taken at least one course in differential equations (e.g., MA 341), and have some basic familiarity with PDEs. Background in numerical linear algebra (e.g., at the level of MA 580) is desirable. However, the required background in numerical methods will be covered as needed, albeit quickly. Also, the required background material on nonlinear optimization, variational methods, and basic probability will be covered throughout the course. Generally, a mathematically mature student will be able to acquire from the lectures the necessary mathematical and computational background. For computational aspects of the course, students should be familiar with programming in either Matlab or Python.

Course topics
Introduction to inverse problems with PDEs; ill-posedness and regularization; variational methods, weak forms; PDE-constrained optimization problems; computing derivatives via adjoints for steady and unsteady PDE problems; descent methods from nonlinear optimization, Newton-conjugate gradient method; Bayesian approach to inverse problems.
Useful references There is no required textbook, but several good references for variational inverse problems (which go far beyond the material covered in class) include:

- **Theory and computational methods for inverse problems:**
  - Curtis R. Vogel, Computational Methods for Inverse Problems, SIAM, 2002. (focus on numerical solution methods, applications mainly in image restoration)

- **Numerical optimization background:**
  - C. Tim Kelley, Iterative Methods of Optimization, SIAM, 1999. (unconstrained, lots of practical advice, PDF is available online)

- **Optimization of systems governed by PDEs:**
  - Max D. Gunzburger, Perspectives in Flow Control and Optimization, SIAM, 2003. (more general than implied by title)

- **Probabilistic approach to inverse problems:**

Academic Integrity Statement and Academic Dishonesty
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"Academic dishonesty is the giving, taking, or presenting of information or material by a student that unethically or fraudulently aids oneself or another on any work which is to be considered in the determination of a grade or the completion of academic requirements or the enhancement of that student’s record or academic career." (NCSU Code of Student Conduct)

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