Course Syllabus

Course Description

This is a graduate level class on Monte Carlo simulation: a statistical sampling technique that uses the power of computers to study complex stochastic systems when analytical or numerical techniques do not suffice. Course topics include random number generator, techniques of generating random objects, pricing of path dependent financial derivatives, estimation of Greeks, design and implementation of discrete-event simulation experiments, output analysis, variance reduction techniques, estimation of steady-state performance, Markov-chain Monte Carlo, simulation optimization. Techniques introduced in this class have strong applications in realistic problems in financial mathematics.

This course is not based on a particular software (such as Arena, Simio and R). Simulation algorithms will be taught in the format of pseudo codes. Students will design and implement relevant simulation procedures in a script-based programming language, such as MatLab or Python.

Time and Place

Monday and Wednesday 3:00–4:15. Room: SAS 2229.

Instructor

Yunan Liu
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Teaching Assistant

Yining Huang
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Textbooks

(i) S. M. Ross, *Simulation*. 5th Edition, Academic Press, 2014. (Required)

(ii) P. Glasserman, *Monte Carlo Methods in Financial Engineering*. Springer, 2003. (Recommended)

Prerequisites

This course is intended for graduate students in mathematics, financial engineering, operations research and related fields. Student are expected to

- have completed a first course on probability theory, statistics, and stochastic models (e.g., MA421, MA546, ST501);

- have fundamental knowledge of financial mathematics (e.g., MA547);
- have knowledge of a programming language such as MatLab and Python.

Homework

There will be weakly homework assignments. Graded assignments will be returned in class.

- Students are encouraged to collaborate with other students in the class, as long as each person writes his/her own solutions and codes.
- But any such collaboration should be clearly **noted** (If some ideas of your solutions come from the discussion with another person, write his/her name on your solution).
- Copying homework from another student (past or present) is **forbidden**.
- Late homework will **NOT** be accepted.

Exams

All exams are closed book and notes. You are allowed to bring a one-page cheat sheet.

- Midterm: March 18 (Monday), temporary.
- Final: April 29 (Monday).

Project

The group project has both modeling and coding components. Each group will be composed of at most three students and will be responsible for

- choosing a topic (after the midterm);
- submitting a project report (by the last day of class);
- giving a project presentation (during the last week of class).

Potential project topics include (but not limited to)

- insurance models,
- pricing and sensitivity analysis of financial derivatives,
- online trading systems,
- service systems (e.g., banks, gyms, call centers, supermarkets, restaurants),
- health care (e.g., hospitals, clinics),
- communication and social networks (e.g., facebook, twitter),
- sports, etc.

Grading

Define the following random variables:
$HW \equiv$ homework, $M \equiv$ midterm, $F \equiv$ final exam, $P \equiv$ project and $G \equiv$ overall grade.
Then the overall grade is given by

$$G \equiv HW \times 20\% + M \times 30\% + P \times 15\% + F \times 35\%.$$

Tentative Course Topics

1. Introduction to simulation
   - Discrete event simulation
   - Monte Carlo simulation
2. Review of basic probability and statistics
   - Probability space
   - Random variables and their properties
   - Estimation of means, variances, and correlations
   - The strong law of large numbers and central limit theorems
   - Confidence intervals and hypothesis tests for the mean
3. Random number generators and numerical integration
4. Generating copies of random variables
   - Inverse transform, acceptance-rejection, composition
   - Generating discrete random variables:
     (i) geometric; (ii) binomial; (iii) Poisson; (iv) discrete uniform.
   - Generating continuous random variables
     (i) exponential; (ii)uniform; (iii) Erlang; (iv) Gamma; (v) Gaussian
   - Dependent random variables and copulas
5. Generating paths of stochastic processes
   - Poisson process: homogeneous, nonhomogeneous and compound
   - Random recursions
   - Continuous- and discrete-time Markov chains
   - Brownian motions
   - Option pricing (European, Asian and American options)
   - Estimation of Greeks
   - Jump diffusion models
6. Simulation via discrete events
   - A single-server queueing system

- A Uber transportation model
- An inventory model
- A production model
- An insurance risk model
- A machine repair problem
- A bitcoin blockchain model
- A limit order book model

7. Output data analysis
    - Transient and steady-state behavior of a stochastic process
    - Statistical analysis for terminating simulations
    - Statistical analysis for steady-state parameters
8. Variance reduction techniques
    - Antithetic variables
    - Control variates
    - Variance reduction by conditioning
    - Stratified sampling
    - Importance sampling
    - Common random numbers
9. Selecting input probability distributions
    - Sample independence
    - Hypothesizing families of distributions
    - Estimation of parameters
    - Goodness of fit tests
10. Advanced topics (dependent on time)
    - Markov-chain Monte Carlo
    - Simulation optimization
    - Rare event simulation